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This study introduces an analytical formalism to shape the quantification of photonic modes devoted to global 3-layer-tubular waveguides (including hollow cylindrical structures). Thereby we have implemented such an overall frame and focus on the asymptotic behavior and the cut-off limit discussions.

Introduction

Owing to their high aspect ratio, sub-wavelength nanotubes are promising building blocks for extreme miniaturization of optical components [1, 2]. In this study, we present a generalized formalism for the whole optical modes of 3-layer-tubular waveguides based on the exact solution of Maxwell's equations. The tubular configuration of interest consists of two regions of infinite length with refractive index n_1 and n_2 embedded in a third infinite media of refractive index n_3 ($n_2 > n_1$ and $n_2 > n_3$). The internal and external radius are respectively noted a and b.

Results

In each region, the six field components have been deduced. By using the boundary conditions, two characteristics equations have been achieved (1). These equations contain two unknown ratios, a_4/a_2 and b_4/b_2, that are obtained by applying Cramer's rule to the eight equations of the boundary conditions. Propagation constants for hybrid modes are obtained by solving simultaneously the two characteristic equations (1) with the proper forms of a_4/a_2 and b_4/b_2.

$$\left\{\begin{aligned}
\left(\frac{\beta\nu}{k_0 n_1}\right)^2\left(\frac{1}{p_{11}^2}+\frac{1}{p_{21}^2}\right)^2 &= \left(\frac{\nu}{p_{11}^2}+\frac{I_{\nu+1}(p_{11})}{p_{11}I_\nu(p_{11})}+\frac{\frac{\nu}{p_{21}}J_\nu(p_{21})-J_{\nu+1}(p_{21})+\frac{b_4}{b_2}\left(\frac{\nu}{p_{21}}Y_\nu(p_{21})-Y_{\nu+1}(p_{21})\right)}{p_{21}\left(J_\nu(p_{21})+\frac{b_4}{b_2}Y_\nu(p_{21})\right)}\right)\times \\
&\quad\left(\frac{\nu}{p_{11}^2}+\frac{I_{\nu+1}(p_{11})}{p_{11}I_\nu(p_{11})}+\frac{n_2^2}{n_1^2}\frac{\frac{\nu}{p_{21}}J_\nu(p_{21})-J_{\nu+1}(p_{21})+\frac{a_4}{a_2}\left(\frac{\nu}{p_{21}}Y_\nu(p_{21})-Y_{\nu+1}(p_{21})\right)}{p_{21}\left(J_\nu(p_{21})+\frac{a_4}{a_2}Y_\nu(p_{21})\right)}\right) \\
\left(\frac{\beta\nu}{k_0 n_3}\right)^2\left(\frac{1}{p_{22}^2}+\frac{1}{p_{32}^2}\right)^2 &= \left(\frac{\nu}{p_{32}^2}-\frac{K_{\nu+1}(p_{32})}{p_{32}K_\nu(p_{32})}+\frac{\frac{\nu}{p_{22}}J_\nu(p_{22})-J_{\nu+1}(p_{22})+\frac{b_4}{b_2}\left(\frac{\nu}{p_{22}}Y_\nu(p_{22})-Y_{\nu+1}(p_{22})\right)}{p_{22}\left(J_\nu(p_{22})+\frac{b_4}{b_2}Y_\nu(p_{22})\right)}\right)\times \\
&\quad\left(\frac{\nu}{p_{32}^2}-\frac{K_{\nu+1}(p_{32})}{p_{32}K_\nu(p_{32})}+\frac{n_2^2}{n_3^2}\frac{\frac{\nu}{p_{22}}J_\nu(p_{22})-J_{\nu+1}(p_{22})+\frac{a_4}{a_2}\left(\frac{\nu}{p_{22}}Y_\nu(p_{22})-Y_{\nu+1}(p_{22})\right)}{p_{22}\left(J_\nu(p_{22})+\frac{a_4}{a_2}Y_\nu(p_{22})\right)}\right)
\end{aligned}\right. \tag{1}$$

The quantification of families of optical modes have been implemented in the case of silica nanotubes ($n_1 = n_3 = 1$, $n_2 = 1.5$ at $\lambda = 670nm$). Two studies of such eigenvalue quantifications are depicted: 1. the internal radius a is fixed and the external radius b ranges around $[a, 10a]$, with $a = [\lambda/10;\ \lambda/2;\ \lambda]$; 2. the external radius b is fixed and the internal radius a ranges around $[0, b]$, with $b = [\lambda/2;\ \lambda;\ 10\lambda]$. The opposite of the effective refractive index has been plotted and studied as a function of the radius (internal or external) normalized to the wavelength. The asymptotic behavior of the curves are particularly discussed as well as the cutoff limits of the optical modes. A map of the different areas (monomode and multimode) has been dressed as a function of the internal and external radius normalized to the wavelength for versatile tubular structures.

References

[1] D. Duval, et al., *Electron. Lett.*, **44**, 1135–1136 (2008).
[2] E. Pouget, et al., *Nature*, **6**, 899–900 (2007).